The effects of party competition on consensus formation

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Abstract

The fight over setting the political agenda is one of the basic mechanisms of party competition of every democracy. However, this political game may have side effects in other aspects of the public debate. One aspect of general interest is how it may alter consensus formation processes among citizens, which may result in states of consensus, polarisation, or opinion fragmentation in the population. In this paper, we study the interrelated dynamics of two processes affecting opinion dynamics when multiple issues are debated. First, we model party competition via campaigning and its effect on the *saliency* or importance with which citizens perceive different political issues. Second, we consider a bounded–confidence model to describe the dynamics of citizens' opinion and consensus formation. We find that the effects of party competition on consensus formation are rich and non-trivially dependent on the configuration of party positions in the political space. We illustrate that —as one would intuitively expect— there are party configurations that foster a paradigmatic state of polarisation for a wide range of model parameters. However, we also show that other party configurations have the opposite effect, and can facilitate reaching a consensus state that could otherwise not have been achieved. Our results illustrate the richness of possible outcomes of interrelations between party competition and consensus formation.

Introduction

Opinion polarisation has been a major concern in modern democracies over the last years, and is even considered a threat to their stability by some authors (Abramowitz and Saunders, 2008; Hare and Poole, 2014; Ramos et al., 2015). Although moderate levels of polarisation have been positively associated with some indicators of democratic performance such as turnout (Wilford, 2017), too much polarisation can lead to social fragmentation, political division, and the inability to reach any compromise between the parties (Somer and McCoy, 2018). In countries such as the US, recent studies show that polarisation has become a major force in shaping voters' attitudes towards policy issues and parties (Campbell, 2018; Svolik, 2019).

In this context, researchers are interested to find out about the role that political actors (Bischof and Wagner, 2019; Hegselmann and Krause, 2015), social media (Sikder et al., 2020; Del Vicario et al., 2017), or external agents (Bhat and Redner, 2019; Gaitonde et al., 2020) have played in splitting societies' positions. Evidently, the need for finding the causes (and ascertaining the remedies) of polarisation has caught the attention of many researchers from political science (Layman et al., 2006; Boxell et al., 2017) and other branches of sociology (Schweighofer et al., 2020).

One of the research fields that provided early contributions to the question of polarisation are physics-inspired studies of opinion dynamics; see Castellano et al. (2009) for a review. Literature in this field typically investigates how micro-behaviours of influence on individuals' neighbours may lead to different macro-properties of the population (e.g., the degree of consensus or polarisation of a society). More specifically, individuals typically hold an opinion (which is modelled as a continuous or discrete variable) that changes upon interaction with other individuals based on rules that vary across different models, but typically consider some form of pressure towards peer alignment.

Among the models and micro–foundations employed in the field, the *bounded–confidence* (BC) model has a long tradition in the study of consensus and polarisation (Ramos et al., 2015; Deffuant et al., 2002). The BC model was conceived in parallel by Hegselmann and Krause (2002) and Weisbuch and Deffuant (Weisbuch et al., 2003), with the introduction of a *tolerance threshold* in opinion difference beyond which interactions do not occur. This mechanism is inspired by the idea that individuals whose opinions are *too different* may never listen to —or, even more, convince— each other. Crucially, the value of the tolerance threshold strongly influences the state of the system in the steady–state, with outcomes ranging from consensus to fragmentation or polarisation (Weisbuch et al., 2003). Since its conception, many other variations of the model have been studied, such as the inclusion of noise (Carro et al., 2013) or network dynamics (Brede, 2019).

Aside from polarisation, our other focus in this work is on the effects of party competition on voters. In political science, most research on party competition originates from spatial voting theory as introduced by Downs (1957), where citizens hold political positions on a multidimensional political space and vote for parties according to the distance of their positions to party positions. In this context, party competition is mainly investigated from two different perspectives. In the first, parties are considered to compete for votes by *moving* towards positions that can attract larger vote shares (Laver and Sergenti, 2011; Miller and Stadler, 1998; Adams and Merril, 1999; Alvarez et al., 2000). In the second, parties compete for vote shares by *adjusting the saliency* given to the different political issues (e.g. by campaigning) to promote aspects of the debate in which they hold favourable positions relative to the electorate (Dragu and Fan, 2016; Amorós and Puy, 2010; Feld et al., 2014; De Sio and Weber, 2014).

In this paper, we unite two disjoint branches of research by studying how party competition interferes with processes of consensus–formation. More specifically, we investigate a model of party competition via saliency adjusting and couple its dynamics to a model of consensus–formation and polarisation via the bounded–confidence model. We discover that the effects of party competition on consensus formation may be very different depending on party positions. Below, we illustrate that party competition can foster consensus formation in some scenarios, but may also promote polarisation in others. We find that these differences in outcomes are generally robust to model parameters and are mainly dependent on the constellation of party positions. Our results serve to emphasise the importance that absolute and relative party positions have in the creation of polarisation.

The model

The purpose of our paper is to investigate interactions between party dynamics and opinion dynamics in the formation of consensus within a society. First, we consider a population of "citizens" or voters that hold individual political opinions modelled as vectors in a multidimensional opinion space. These opinions evolve subject to peer interactions in the population following the well-known bounded-confidence model (Weisbuch et al., 2003). Second, we also consider a set of parties each of which holds a fixed opinion. Parties compete for votes by adjusting the emphasis with which they promote various issues (represented by dimensions) in the opinion space. In other words, they affect the perceived importance or *saliency* of the dimensions.

In more detail, we assume that citizens cast their vote in a probabilistic fashion according to their perceived distance to the parties (Burden, 1997; Alvarez and Nagler, 1998). Parties then compete by promoting issues and thus changing the perception of distances in the population. Party competition and consensus formation in the population interact, as the consensus dynamics is affected by perceived distances between citizens' positions. We proceed by giving a detailed explanation of the modelling of opinion and party dynamics below.

Opinion dynamics model

Consider a population of citizens whose opinions are initially uniformly spread on a D-dimensional space of opinions constrained to the interval $[-1, 1]$, $O : [-1, 1]^D$. Citizens $i = 1, \dots, N$ hold opinions $\boldsymbol{o}_{i,t} \in O$ that change over time $t = 0, \dots, T$. Opinion dynamics follow the *bounded-confidence* (BC) model introduced by Weisbuch et al. (2003), as it will be explained in the following. At each time step t, two randomly chosen citizens i and j interact and converge in their opinions provided that the distance between their opinions is less than a specified tolerance threshold $\delta \in [0, 1]$; otherwise, their interaction has no effect. Formally,

$$\boldsymbol{o}_{i,t+1} = \begin{cases} (1-\alpha)\,\boldsymbol{o}_{i,t} + \alpha\,\boldsymbol{o}_{j,t} + \boldsymbol{\eta} & \text{if } \mathrm{d}_{i,j}^{(t)} \le \delta \\ \boldsymbol{o}_{i,t} + \boldsymbol{\eta} & \text{otherwise,} \end{cases} \quad (1)$$

where $\alpha \in [0, 1/2]$ is a parameter that models how much is conceded upon interaction, $\boldsymbol{\eta}$ is Gaussian white noise $\boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \sigma^2 I)$ representing external influences not captured by the model, and d is a normalised distance metric $\mathrm{d} : O \times O \to [0, 1]$, with $\mathrm{d}_{i,j}^{(t)} = \mathrm{d}(\boldsymbol{o}_{i,t}, \boldsymbol{o}_{j,t})$. Note that the updated opinion $o_{i,t+1}$ needs to be truncated to the interval $[-1, 1]$ in all dimensions, in case the noise term moved it outside this region. Citizen j also simultaneously updates her opinion $o_{j,t+1}$ in an analogous move.

We assume that perceived distances in opinion are affected by the perception of the importance of the various dimensions, represented by a normalised vector of saliencies $w_d \in [0, 1], d = 1, ..., D$, that sums up to 1. Accordingly, we employ a weighted Euclidean distance given by

$$\mathrm{d}(\boldsymbol{o}_i, \boldsymbol{o}_j) = \frac{1}{2}\sqrt{\sum_{d=1}^{D} w_d \left(o_i^{(d)} - o_j^{(d)}\right)^2}\ . \quad (2)$$

Party competition

On top of the population of voting citizens, we also consider a number of parties $k = 1, \dots, K$ that have fixed opinions $\boldsymbol{p}_k \in O$. According to probabilistic models of spatial voting (Burden, 1997; Alvarez and Nagler, 1998), party k is assumed to receive a vote from citizen i at time t, i.e. $v_{i,t} = k$, with a probability that is related to the distance of their respective opinions and the distance of the citizen's opinion to the other parties:

$$P(v_{i,t} = k) = \frac{\exp(-\gamma\,\mathrm{d}(\boldsymbol{o}_{i,t}, \boldsymbol{p}_k))}{\sum_{l=1}^{K} \exp(-\gamma\,\mathrm{d}(\boldsymbol{o}_{i,t}, \boldsymbol{p}_l))}\ , \quad (3)$$

where γ parameterises the range of party influence. For large γ, parties have a small range of attraction around their core positions in opinion space; for small γ, parties can attract voters with more strongly deviating opinions[1].

[1]Note that with $\gamma \to \infty$, a deterministic model of voting is recovered, where votes are always given to the party whose position

We assume that party k has a vector of preferred saliencies $\boldsymbol{w}^{(k)} = (w_1^{(k)}, \ldots, w_D^{(k)})$ normalised to $\sum_{d=1}^{D} w_d^{(k)} = 1$, as to model a resource constraint in party campaigning. Following ideas of Amorós and Puy (2010) and Dragu and Fan (2016), citizens perceive the overall effect of all parties' campaigning efforts —which determines the overall saliency $\boldsymbol{w}$ of issues in the population— through $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{w}^{(1)}, \ldots, \boldsymbol{w}^{(K)})$. The function $\boldsymbol{f} : [0,1]^{D \times K} \to [0,1]^D$ is assumed to be differentiable and monotonic, and produces normalised output. For the purposes of this study, we consider the simple aggregation function $f_d(\boldsymbol{w}^{(1)}, \ldots, \boldsymbol{w}^{(K)}) = 1/K \sum_{k=1}^{K} w_d^{(k)}$.

Party k attempts to maximise its expected vote share from the electorate at each time step by directing attention to issues beneficial to gaining votes, i.e. by adjusting its vector of preferred saliencies $\boldsymbol{w}_t^{(k)}$ through campaigning. The expected vote share of party k at time t is defined as $V_{k,t} = 1/N \sum_{i=1}^{N} P(v_{i,t} = k)$. In the following, we consider parties to be myopic; i.e. we assume they attempt to maximise their vote share at the current time, $V_{k,t}$, not taking into account the vote share at a future time $t' > t$ and not accounting for the strategy of the opponents or the dynamics of opinions. We assume that parties adjust their preferred saliencies at a time scale τ, modelled by implementing changes in preferred saliency after every τ time steps.

To model rational parties that have local knowledge of the effects of changing their campaigning efforts, we assume that parties follow a gradient dynamics, adjusting their saliencies in the direction of largest (expected) vote share gain $\nabla_{\boldsymbol{w}^{(k)}} V_{k,t}$, which is given by

$$\nabla_{\boldsymbol{w}^{(k)}} V_{k,t} = 1/N \sum_{i=1}^{N} \nabla_{\boldsymbol{w}^{(k)}} P(v_{i,t} = k) =$$

$$= \gamma/N \sum_{i=1}^{N} P_{i,k}^{(t)} \left(\sum_{l=1}^{K} P_{i,l}^{(t)} \nabla_{\boldsymbol{w}^{(k)}} \mathrm{d}_{i,l}^{(t)} - \nabla_{\boldsymbol{w}^{(k)}} \mathrm{d}_{i,k}^{(t)} \right) , \quad (4)$$

where $P_{i,k}^{(t)} = P(v_{i,t} = k)$ and $\mathrm{d}_{i,k}^{(t)} = \mathrm{d}(\boldsymbol{o}_{i,t}, \boldsymbol{p}_k)$. For the aggregation function f_d introduced above, we have

$$\frac{\partial \mathrm{d}_{i,l}}{\partial w_d^{(k)}} = \frac{1}{2K} \left(o_i^{(d)} - p_l^{(d)} \right)^2 \mathrm{d}_{i,l}^{-1} . \quad (5)$$

Specifically, after every τ updates of the bounded–confidence dynamics of opinion formation, parties adjust saliencies via

$$\boldsymbol{w}_{t+1}^{(k)} = \boldsymbol{w}_t^{(k)} + \epsilon/\gamma \, \nabla_{\boldsymbol{w}^{(k)}} V_{k,t} , \quad (6)$$

which is then projected into the feasible region $\sum_{d=1}^{D} w_d^{(k)} = 1$ and $w_d^{(k)} \in [0,1]$. In the above, we introduce the parameter $\epsilon \in \mathbb{R}_{0+}$ that allows to adjust step size and models the timescale at which parties' salience adjustments become effective[2].

We conclude this section with Table 1, which summarises all parameters introduced in the model.

Parameter	Definition
$N, D, K \in \mathbb{N}$	N. of citizens, dimensions, parties
$\delta \in [0,1]$	BC tolerance threshold
$\alpha \in [0, 1/2]$	Concession rate upon interaction
$\sigma \in \mathbb{R}_{0+}$	Standard deviation of Gaussian noise
$\gamma \in \mathbb{R}_{+}$	Range of party influence
$\tau \in \mathbb{N}$	Saliency–changing period
$\epsilon \in \mathbb{R}_{0+}$	Speed of saliency change

Table 1: Parameters of the model.

Results

Below, we explore the model and its outcomes via agent–based simulations, as analytical solutions are hard to obtain due to the non-linearities introduced in the BC model. We run each simulation until a pseudo–equilibrium has been reached, i.e. until the difference in the standard deviation of citizens' opinion in the last 50τ time steps is less than 10^{-4}, i.e. $\mathrm{std}(\boldsymbol{o}_{t-50\tau}) - \mathrm{std}(\boldsymbol{o}_t) < 10^{-4}$. For simplicity, we focus on the case of $D = 2$ dimensions and $K = 3$ political parties, which already serves to illustrate the richness of possible model outcomes. Investigation of other scenarios is reserved for future work.

Following previous work on the BC model (Lorenz, 2007), we focus our analysis on the number of clusters in the steady–state. However, instead of just considering the number of clusters (which might be strongly influenced by the occurrence of some very small clusters), we employ the *effective number of clusters*, as given by $\phi = 1/\sum_{c \in C}(s_c/N)^2$, where s_c is the size of cluster c and C the set of all clusters found. With this measure, consensus corresponds to $\phi \sim 1$ and paradigmatic (maximum) polarisation to $\phi \sim 2$, meaning that the population is divided into two clusters of similar size. Values of ϕ between 1 and 2 generally entail the presence of two clusters of unequal size, and $\phi > 2$ corresponds to fragmentation into more than two clusters. For practical implementation, to identify clusters we use the algorithm DBSCAN (Daszykowski and Walczak, 2009) with a radius of δ.

The consensus effect of party competition

For the purpose of illustration, we examine in detail a single simulation run to obtain basic intuition of how the intertwined opinion and party dynamics may affect each other.

is closest to the citizen. Lower values of γ imply the notion of uncertainty in choice, as there may be hidden attributes not modelled in voting space or voters may not vote for *what is best for them*. Similar forms to Eq. (3) are common in choice theory (Anderson et al., 1992; McFadden, 1994)

[2]Note that we have introduced two parameters (ϵ and τ) related to the time scale in which parties adjust their saliency strategies. The purpose of introducing these redundant parameters is to limit computations of gradients and improve computational efficiency.

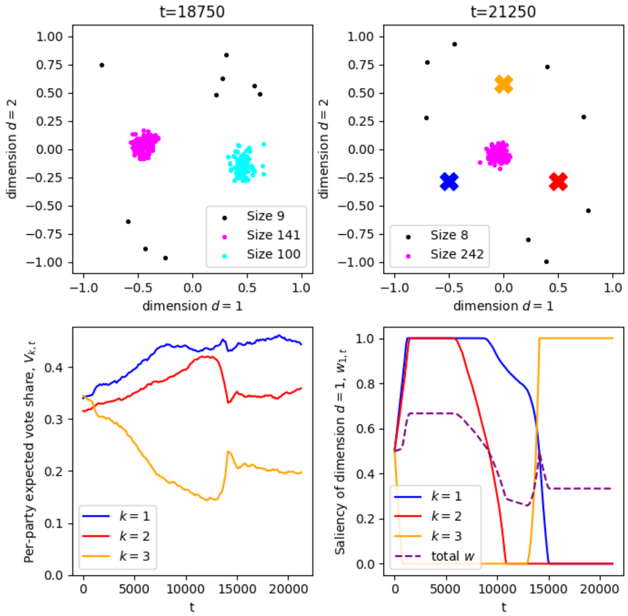

Figure 1: Illustration of results from a single simulation run. The first row shows the stationary outcome for runs without (*left*) and with (*right*) party competition. Opinions are coloured by the cluster they belong to, while the legend shows cluster sizes. The bottom row shows the evolution of each party's vote share (*left*), and parties' saliency mix (*right*). Party positions are $\boldsymbol{p}_{k=1} = \left(-1/2, -1/2\sqrt{3}\right)$, $\boldsymbol{p}_{k=2} = \left(1/2, -1/2\sqrt{3}\right)$, and $\boldsymbol{p}_{k=3} = \left(0, 1/\sqrt{3}\right)$. The parameters used are $N = 250$, $\delta = 1/5$, $\alpha = 1/4$, $\sigma = 1/40$, $\gamma = 10$, $\tau = N/2$, and $\epsilon = 32$. Note that both scenarios share the same seed for the pseudo–random number generator, so the initial opinion distribution and the random pattern of agent selection for interaction is identical in both scenarios.

For this illustrative simulation, we place parties forming an equilateral triangle with a side length of 1 centred in the opinion space. We set the number of citizens to $N = 250$ and scatter them with uniform probability in the opinion space O. Parties initially start with a neutral saliency mix; i.e. they promote all issues with equal strength, $w^{(k)}_{1,t=0} = w^{(k)}_{2,t=0} = 1/2$.

The top–left panel of Fig 1 illustrates the stationary outcomes of voter distributions of a simulation where there is no party competition, i.e. the saliency mix remains equal for both dimensions for the whole run. The top–right panel of the figure illustrates the stationary outcome of a simulation where parties compete to increase their vote share. Coloured crosses indicate the positions of the parties. The second row of Fig 1 gives the evolution of different metrics over time for the competitive scenario. The left panel shows parties' vote share, and the right panel shows parties' preferred saliencies.

In the simulation of a standard BC model without party competition (top–left panel of Fig 1), it can be seen that opinions gather into two clusters of similar sizes (see legend). In contrast, the top–right panel shows a scenario with party competition that leads to an outcome of consensus on a single cluster. From the bottom–right panel, it can be seen that, early after the simulation starts, the aggregated saliency mix (*dashed*) is tilted towards dimension $d = 1$, as parties $k = 1, 2$ initially favour it. This initial promotion of dimension $d = 1$ makes citizens consider the dimension more thoroughly when perceiving distances to other citizens, leading to two clusters aligned along dimension $d = 1$. This outcome can be understood from the changes in saliency likened to deformation of the opinion space. When one dimension from the saliency mix is favoured, the opinion space 'elongates' in this dimension and 'shrinks' in the other dimensions, so clusters align along the favoured dimension. However, around $t \sim 7000$, party $k = 2$ shifts its saliency promotion to fully promoting dimension $d = 2$, which is followed by party $k = 3$. Since $d = 2$ becomes now predominant and most citizens already have similar opinions in this dimension, the two clusters unite and lead to consensus of the population.

Systematic exploration of the effect of party competition on consensus forming

While the previous preliminary exploration allows us to understand one of the basic mechanisms present in the proposed model, it is restricted to a single run. One may wonder how robust the observed effect is with respect to the chosen parameters and the stochasticity of the model. For instance, the tolerance threshold δ is known to have a crucial role in consensus formation (Lorenz, 2007). From previous literature, we know that low values of δ typically result in the formation of many distinct —or *fragmentation*—; intermediate values of δ are known to lead to a state of polarisation with two camps of similar size; and high values typically entail consensus (Hegselmann and Krause, 2002; Weisbuch et al., 2003; Lorenz, 2007). The noise parameter σ is also of relevance for determining model outcomes. Not only does it determine the variance of opinions within clusters, but also the minimum gap between clusters that guarantees that they do not unite, at least in the short run.

Fig 2 investigates the effective number of clusters ϕ for different values of noise σ and tolerance threshold δ. Let us first focus on the case without party competition. On the left panel, with $\delta = 0.18$, the system reaches a state of fragmentation ($\phi > 3$) for low values of σ. As σ increases, the effective number of clusters ϕ decreases as a result of noisy moves that may bring clusters together. A similar trend can be observed when raising the tolerance threshold to $\delta = 0.2$ (middle panel), although shifted to smaller ϕ. With $\delta = 0.22$ consensus always results, even when there are no noisy moves ($\sigma = 0$).

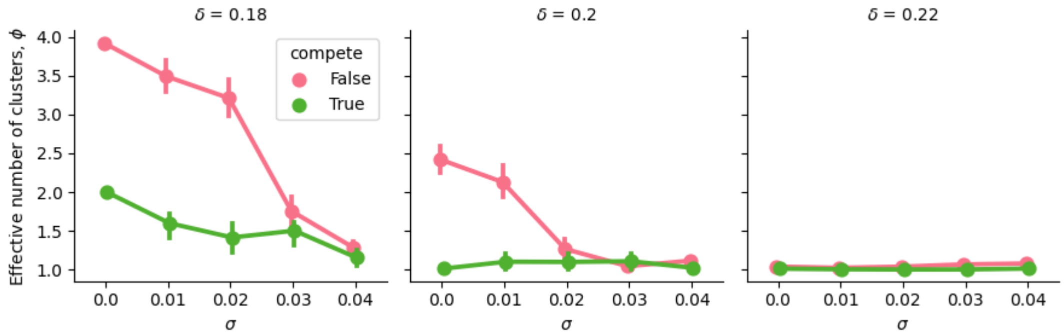

Figure 2: Dependence of the effective number of clusters ϕ on the tolerance threshold δ and the strength of noise σ for parties positioned in a triangle shape, as in Fig 1. Error bars are standard errors of the mean estimated from 10 simulations and other parameters are $\gamma = 10$, $\tau = N/2$, $\alpha = 1/4$, $N = 2500$, $\epsilon = 32$.

Next, consider the case of party competition. For this scenario, when $\delta = 0.18$ (left panel), consistently a smaller effective number of clusters results when compared to the case of no party competition. When $\delta > 0.2$, consensus consistently results regardless of the level of noise σ.

In summary, the scenario of party competition laid out above fosters consensus, resulting in populations with smaller effective numbers of clusters than when no party competition is present. These results highlight that what has been shown in the illustrative simulation run is indeed a typical outcome for this configuration of party positions.

Exploring other party configurations

For the specific party configuration seen above, we have seen that party competition can influence consensus dynamics, and promote consensus in the population. Here, we illustrate that, for other party configurations, very different effects can emerge from party competition. More specifically, we will show another party layout with which one may observe the opposite effect and see that party competition can also *create polarisation*. An example of this can be seen in simulation results illustrated in Fig 3. The scenario analysed here has identical parameters to those of Fig 1, except for the party layout and the initial distribution of opinions.

Simulation results in Fig 3 are laid out in analogy to Fig 1. In the top–left panel of Fig 3, we again see the stationary outcome of the model without party competition, leading in this case to consensus with a single cluster. In contrast, the scenario including party competition (top–right) now drives the population to a state of polarisation, with one cluster slightly bigger than the other (see the legend). As can be seen from the bottom–right panel, for this scenario parties favour one of the dimensions at the beginning of the simulation and stick to that decision for the remaining time.

This polarisation–promoting side effect is not a result of stochastic fluctuations in the scenario shown in Fig 3. In Fig 4, we follow up on the presented initial results and cal-

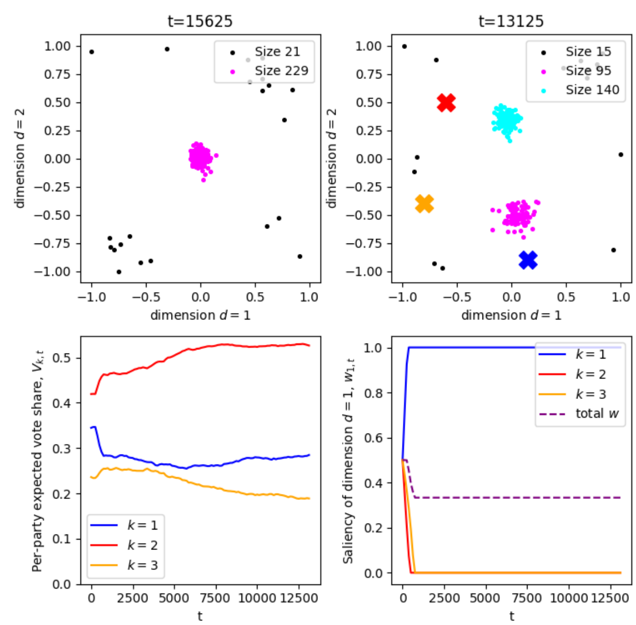

Figure 3: Exploratory run with an alternative party layout. The first row shows the stationary outcome of citizen opinions for runs without (*left*) and with (*right*) party competition. Opinions are coloured by the cluster they belong to, while the legend shows cluster sizes. The bottom row shows the evolution of each party's vote share (*left*), and parties' saliency mix (*right*). Party positions for this experiment are $\boldsymbol{p}_{k=1} = (0.15, -0.9)$, $\boldsymbol{p}_{k=2} = (-0.6, 0.5)$, and $\boldsymbol{p}_{k=3} = (-0.8, -0.4)$.

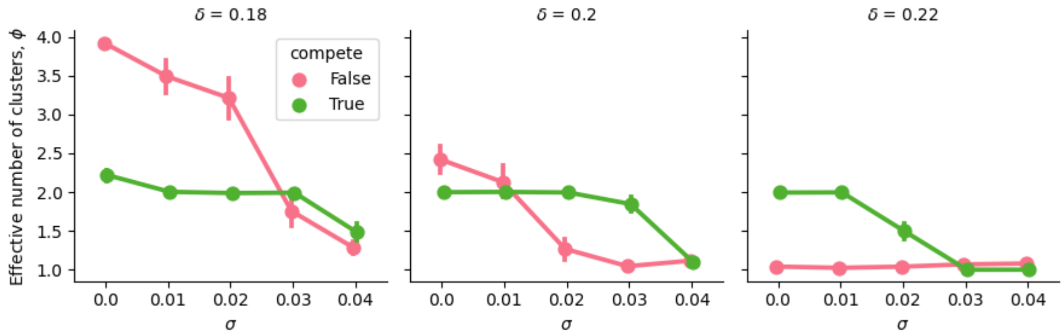

Figure 4: Dependence of the mean effective number of clusters on the BC tolerance threshold δ, the strength of noise σ, and the saliency–changing speed ϵ for the same party layout as in Fig 3. Error bars are standard errors of the mean across 20 simulations that start from different initial distributions of citizen opinions randomly drawn from a uniform distribution over the opinion space O.

culate averages over multiple runs of the opinion dynamics. The figure illustrates the dependence of the mean number of effective clusters on δ and σ for the same party layout as in Fig 3. For this scenario, when $\delta <= 0.2$ (left and middle panels) and party competition is present, a paradigmatic polarisation with $\phi \sim 2$ is consistently present for all σ values up to 0.04. When increasing δ to 0.22, paradigmatic polarisation only occurs for values of σ up to 0.01. This contrasts with consistent consensus outcomes achieved by competition–free simulations for $\delta = 0.22$.

Our systematic exploration of these two scenarios with different party layouts illustrates that including considerations of the effects of party competition in models of opinion dynamics can both favour and prevent consensus.

Summary and future work

In this paper, we have studied the interrelation of dynamics of party competition via saliency adjustment and opinion dynamics in the form of a bounded–confidence model. We have explored different parameter regimes and have pointed to two very different effects of party competition that depend on party layout.

In the first case, party competition fosters consensus. This happens due to parties alternating saliency promotion for different issues across the simulation. Such behaviour initially creates division along one political issue, and then takes away importance from the issue, allowing the two clusters of opinion to merge. In contrast, in the second case, party competition leads to the enhancement of polarisation in the society. This outcome results from parties putting more emphasis on a single issue for the whole period of time, making the electorate more divided along this issue.

Despite the interest these results elicit, several questions remain unanswered and should be addressed in future work. First and most importantly, the characteristic of the party constellation that causes the promotion of consensus as opposed to polarisation remains to be determined. Second, for the sake of simplicity, we have studied a simple scenario with only three parties and two dimensions. The effects of increasing any of these should be assessed, as there are many real cases where these numbers differ from the ones explored here. Third, we have made several choices in the model with potentially strong impacts on the results. For instance, we have assumed the distance metric to be Euclidean instead of, e.g., a city–block distance. Such model variations should be explored. Last, the model admits many other variations and extra modifications. Some of them can be of interest, such as other implementations of the saliency–aggregation function —e.g. giving different power to influence saliencies to parties according to their previously achieved vote share—; or considering abstention in voting when a citizen's position has no party position nearby.

In conclusion, this work explores a relatively new research question that combines ideas of campaigning competition between political parties and consensus formation via opinion dynamics. The results shown here, although preliminary, unveil a rich behaviour of the interaction of the two models. Further investigation is required for a complete understanding of these mechanisms and their consequences in modern societies.

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