

Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis

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Abstract

Although cognitive complexity has been usually related to brain size or number of neurons, they are neither necessary nor sufficient conditions, since there are many species in nature showing that even with simple brains, they can exhibit unexpected levels of behavioral complexity. More recent approaches to cognitive science, such as enactive cognition, have been investigating social interaction in itself as part of an individual's cognition. Recently, Candadai et al. (2019) have demonstrated through a minimal model that social interaction increases agents' neural complexity and revealed that this cannot be achieved in isolation. In this paper, we first replicate the Candadai et al. (2019) model to analyze the state-space of the autonomous continuous-time recurrent neural networks of the interacting agents. Our results show that in terms of complexity, it is as simple as it can be, a single fixed-point attractor. Then, we proceed to ask whether, after loosening up the parameter constraints of this model, we will find more complexity in the state-space as there will be a broader variety of values in the parameters of neural controllers encoded in the genotype of each agent. Surprisingly, the state-space of this second approach leads to the same results, a single fixed-point attractor. Our findings, then, support the idea that cognitive complexity is mainly driven by the dynamics of social interaction rather than internal complexity.

Introduction

Traditionally, cognitive complexity has often been associated with brain size or number of neurons. The social brain hypothesis supports these ideas by assuming that socially living species, such as human and nonhuman primates, should have enlarged brain sizes as a result of the demands of social life, in comparison to the more isolated ones. Briefly, this hypothesis is based on the correlation between social complexity and brain/group sizes (Dunbar, 1998; Barret et al., 2007). In contrast to these ideas, Barret et al. (2007) suggested that cognitive complexity might emerge from the interaction of brain, body and environment, and it is not simply attributable to the level of internal complexity itself. Furthermore, Barret (2011) argued that there are many of the so-called simpler animals with small brains that exhibit striking levels of behavioral complexity, such as paper wasps, which have the ability to recognize facial traits and use it to preserve social order in their hives and ants, which are able to

find the shortest path to food sources through self-organizing processes. Therefore, these examples reinforce the idea that the complexity of behavioral activity is not merely a consequence of internal complexity and they highlight the importance of social interaction.

From an enactive cognition approach, social interaction is defined as a complex phenomenon involving engagement of at least two agents in a complex co-regulated pattern that enables social cognition (De Jaegher et al., 2010). In the last decades, there have been increasing efforts in investigating social interaction by using agent-based models to provide proof of concepts in order to make conceptual advances. Some of these models known as "minimal models" have been inspired by William Grey Walter's "turtle" mobile robots (Walter, 1950). These robots, which had only touch and light sensors and a very simple control architecture, performed surprising complex patterns of behaviors when interacting. On the basis of these findings, Di Paolo (2000) developed a simulation model of acoustically coupled embodied agents to study social coordination. In his model, these agents showed interesting behaviors, such as turn-taking and organized movement that emerged through their interaction via an acoustic medium. On the other hand, Froese et al. (2013a) created a minimal model of two acoustically coupled agents and demonstrated that in interactive scenario the neural dynamics of these agents has formal properties that could not be generated in isolation. Similar minimal approaches have been shown also to be very productive in studies of real social interaction in humans, e.g. perceptual crossing experiments (Froese, 2018).

Based on those previous works and following an evolutionary robotics methodology, Candadai et al. (2019) demonstrated that social interaction increases the complexity of an agent's neural activity and revealed that this cannot be achieved in isolation. They performed experiments on agent-based modeling using pairs of acoustically coupled embodied agents in an empty 2-dimensional environment, as shown in Figure 1.

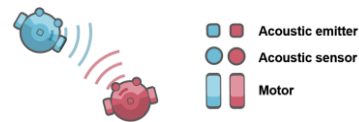


Figure 1: Illustration of a pair of acoustically coupled embodied agents in Candadai et al. (2019) model.

These experiments consisted of measuring neural entropy as an index of internal complexity. This was motivated by its interpretability, computational tractability and because of previous studies which have related higher levels of neural entropy with enhanced cognitive performance, e.g. enhanced generalization in motor learning tasks (Dotov and Froese, 2018). There were two different scenarios: by artificially evolving interacting pairs of embodied agents (maximizing interaction entropy) and by artificially evolving isolated agents in the environment (maximizing isolation entropy). Moreover, they measured the interaction entropy of an agent in the presence of a “ghost” partner, which was playing back pre-recorded behavior of previous trials, thus, they were not able to mutually interact with each other. As a result, this led to a loss of internal complexity of the “live” agent and proved that active interdependent interaction increases their neural complexity.

We found the Candadai et al. (2019) model very insightful, however, they did not provide a dynamical systems analysis to get clarity on the internal complexity, consequently, we got the next open questions: How complex are the internal state spaces of these agents? And how is that internal complexity related to their underlying genetic complexity, and how does it evolve over generations? Therefore, in this study we aim to achieve three novel contributions:

1. **Loosening up genetic constraints:** By proposing a novel approach that we call “layer-based unconstrained”, in order to see the effects of the loosened parameter constraints and allow a broader diversity in the genotypes of the agents.
2. **State-space analysis:** By performing the dynamical systems analysis of the autonomous continuous-time recurrent neural network (CTRNN) of each best pair of agents for both approaches (layer-based constrained (original configuration) and layer-based unconstrained (proposed configuration)) in order to analyze their internal complexity.
3. **Evolutionary analysis of neural entropy:** By observing how the normalized neural entropy of the best pair of agents is evolving through time in two different scenarios: interactive and under “ghost” condition.

Thus, these goals share the following underlying reason for being studied: To demonstrate that social interaction matters and makes a difference in the complexity of neural dynamics.

Methods

We started working on our model as a replication of the Candadai et al. (2019) model. We choose to maximize only interaction entropy. The fitness function for the evolutionary search (i.e. neural entropy, which has been used as an indication of cognitive complexity) does not explicitly optimize social interaction nor does it optimize any specific task. The implementation details of this model and our modifications are presented as follows:

A. Body

The body of each embodied agent is designed as circular with a radius, R , of 4 units, which has two acoustic sensors symmetrically positioned at an angle of 45 degrees to the central axis (i.e. positioned at 45 and 315 degrees, respectively); an acoustic emitter placed on the center of the body, therefore, equidistant to its own sensors; and two motors driving wheels placed on opposite sides of the agent that enable movement in a 2-dimensional environment. This design was initially inspired by Di Paolo’s acoustically coupled agents (Di Paolo, 2000). The strength of the emitted signal experiences linear loss with distance. It will be maximum and equal to that of the emitted strength at a distance $D = 2R$, between the center of the agents and linearly decays with increasing distance. Furthermore, the “self-shadowing” mechanism, i.e. experienced attenuation when the signal travels within the embodied agent, is modeled as a scaling factor over the sensory inputs in a range from 0.1 to 1. The equations for calculating the shielded distance, D_{sh} , that the signal passes through the body can be found in the Supplementary Material of Candadai et al. (2019). Then, the process of obtaining the sensory input for each sensor consists in first calculating it by applying the inverse square law without any “self-shadowing” attenuation, based on the distance between the sensor and the source, and then multiplying by the “self-shadowing” attenuating factor linearly mapped from 1 (when $D_{sh} = 0$) to 0.1 (when $D_{sh} = 2R$).

B. Environment

The simulated environment is a 2-dimensional unlimited arena. Collisions are modeled as point elastic, i.e. no changes in their angular velocity (no friction between bodies) and conserving the momentum of the whole system by having zero net effect on their velocity vectors. This is achieved by exchanging the velocity vectors of the embodied agents, so they simply bounce off each other without loss of energy. Modeling energy transfer is considered for future work.

C. Neural architecture

The neural architecture of each of the embodied agents is composed of three layers, we called them as follows: sensor layer, neuron layer and actuator layer.

1. Sensor layer: The sensor layer consists in two sensor nodes with a sigmoidal activation function. Their output is given by:

$$o_s = g_s \sigma(I_s + \theta_s) \quad (3)$$

where $\sigma(x) = 1/(1 + e^{-x})$ is the sigmoidal activation function, g_s is the sensory gain, I_s is the sensory input and θ_s is the bias.

2. Neuron layer: The neuron layer is modeled as a continuous-time recurrent neural network (CTRNN) (Beer, 1995), consisting in two fully recurrently connected neurons, this corresponds to a 2-dimensional dynamical system. The activity in each neuron is governed by the following state equation:

$$\tau_i \frac{dy_i}{dt} = -y_i + \sum_{j=1}^N w_{ij} \sigma(y_j + \theta_j) + \sum_{s=1}^2 w_{is} o_s \quad (4)$$

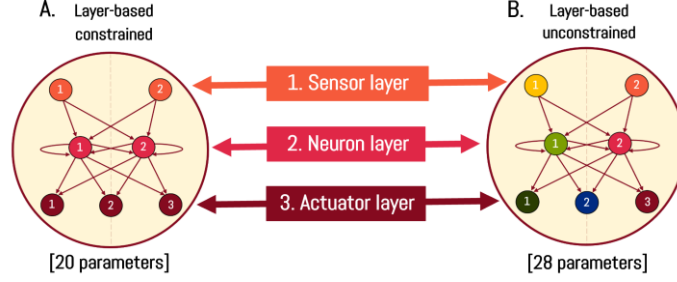


Figure 2: Two approaches based on layer parameter constraints of the neural controllers for the evolutionary search. **(A)** Layer-based constrained. This is the original configuration in the Candadai et al. (2019) model, where for N agents, the genotype contained $20N$ parameters. In this approach, the two acoustic sensor nodes have common gain and bias, the two neurons have common time-constant and bias, and the three actuator nodes (two motors and an acoustic emitter) have common gain and bias. **(B)** Layer-based unconstrained. This is the proposed configuration in order to loosen up the original parameter constraints and allow more diversity in the genotype, where for N agents, the genotype contained $28N$ parameters. In this approach, sensor nodes, neurons and actuator nodes do not have common parameters.

where dy_i/dt refers to the rate of change of internal state y_i of neuron i based on a time constant τ_i . This rate of change depends on three values: the current state, the weighted sum of outputs from the two neurons ($N = 2$) in the network and the total external input. The input from other neuron is calculated by weighting their output with weights from j to i , i.e. w_{ij} . The output of each neuron based on its internal state is given by $\sigma(y_i + \theta_j)$ where θ_j refers to a bias term for that neuron. Lastly, the state is influenced as well by the total external input received by the neuron, given by the weighted sum of the sensory input with weights w_{is} from sensor node s to neuron i and o_s being the sensory output from two sensors.

3. Actuator layer: The neurons feed into the actuator layer, where the input to each actuator node is a weighted sum of the outputs of the neuron. The actuator layer contains three actuator nodes, two corresponding to the left and right motors and one corresponding to the acoustic signal emitter. All of them are sigmoidal units with a gain and bias such that the output of actuator node i , m_i , is given by:

$$m_i = g_m \sigma \left(\sum_{n=1}^N w_{ni} * o_n + \theta_i \right) \quad (5)$$

where o_n is the output of the neuron, that is weighted by w_{ni} and θ_i is the bias term, and g_m is their gain.

Note that locomotion is managed by the effective control of the two motors. Net linear velocity is given by the average of their corresponding outputs and angular velocity which rotates the agent is given by their difference divided by the radius of the agent.

D. Neural entropy

During the entire course of behavior, i.e. 4 trials, the neural activity of each of the agents is recorded. Then, neural entropy in the 2-dimensional time series from the outputs of the two neurons (neuron layer) is measured as the neural complexity. The outputs of the two neurons are bounded in the range $[0,1]$, as they are obtained from a sigmoid function. The output space is binned with 100 bins along each dimension, i.e. each axis corresponds to the outputs of the first and the second

neuron, respectively, each axis goes from 0 to 1 and is divided into 100 bins, totaling 10,000 bins. Thus, a 2-dimensional histogram is created at the end of the 4 trials with all the recorded binning data points. The entropy H of the neural time series is given by:

$$H = \sum_{i=1}^{100} \sum_{j=1}^{100} -p_{ij} \log(p_{ij}) \quad (6)$$

where the probability of the neural activity in a particular bin $[i,j]$, p_{ij} , is given by the number of data points in that bin divided by the total number of data points. The neural entropy then is normalized in the range $[0,1]$ through dividing by the maximum possible entropy, $\log(100 * 100)$, which is obtained when all bins are uniformly populated, therefore, a uniform distribution over the 2-dimensional histogram is achieved. Hence, normalized neural entropy is given by:

$$\hat{H} = H / \log(100 * 100) \quad (7)$$

E. Evolutionary algorithm

A real-valued genetic algorithm was used as an optimization technique for the parameters of the neural controllers in order to maximize the agents' neural entropy. In contrast to the Candadai et al. (2019) model, here we followed two different approaches for the evolutionary optimization, namely, layer-based constrained and layer-based unconstrained, as shown in Figure 2. Each of these approaches is described as follows:

1. Layer-based constrained: This approach followed the original configuration in the Candadai et al. (2019) model, where each agent had 20 parameters, i.e. for N agents, the genotype contained $20N$ parameters. Here, both sensor nodes shared same gain and bias, both neurons shared same time-constant and bias, and the three actuator nodes shared same gain and bias. Therefore, in each of the three layers (sensor layer, neuron layer and actuator layer) the parameters were limited to have common values.

2. Layer-based unconstrained: This is the proposed approach in order to loosen up the identified parameter constraints to allow more diversity in the genotypes and see the possible effects in the subsequently analysis. Here, in each of the three layers (sensor layer, neuron layer and actuator

layer) there were no common parameter values. Thus, each agent had 28 parameters, i.e. for N agents, the genotype contained $28N$ parameters.

In both approaches, each of the parameters were initially encoded in the range $[-1,1]$. When performing the trials to evaluate the performance, these parameters were scaled in different ranges in order to build the agents. For the sensor layer and actuator layer, their gains were scaled in the range $[1,5]$ and their biases were scaled in the range $[-3,3]$. For the neuron layer, its time-constants were set in the range $[1,2]$ and their biases were set in the range $[-3,3]$. Additionally, all weights from the three layers, were scaled in the range $[-8,8]$. All these parameter ranges were the same as those in the Candadai et al. (2019) model.

We performed 10 independent runs for each of the approaches (layer-based constrained and layer-based unconstrained), totaling 20 runs. Each of the runs started with a random population of 96 solutions, where each of them encoded the parameters for two agents in the interactive scenario and were evolved for up to 2000 generations.

In each generation, the agents built from the genotype were evaluated over 4 independent trials. Each trial lasted 200 units of time at a step size of 0.1. At the beginning of the trials, the agents were placed at 20 units from each other but varying their relative angle as $[0, \pi/2, \pi, 3\pi/2]$ for each trial. During the 4 trials, the neural activity of each agent was recorded, and at the end, the normalized neural entropy was calculated, and fitness was set as the average normalized neural entropy of the two agents.

After the performance evaluation, an elite population of the top 4% solutions was kept as is, and the remainder of the solutions for a new population was created by mutating and, then, crossing over this elite fraction. Mutation was applied by adding a zero-mean Gaussian mutation noise with variance 0.1 to the solutions, while, crossover involved that each parameter between a pair of solutions was swapped with a probability of 0.1.

F. Analysis under “ghost” condition

In order to delineate the role played by interdependent interaction on internal complexity, the best pair of agents in each of the selected generations (0, 1, 2, 3, 4, 5, 10, 50, 100, 500, 1000, 2000) were tested under a “ghost” condition. Blue agent was referred as the “ghost” agent and red agent was referred as the “live” agent. The “ghost” agent was replaying pre-recorded behavior from previous trials and the “live” agent was allowed to interact with it. The active agent started at a different random initial angle from “ghost” agent, in order to not repeat its behavior from those trials, while keeping the initial distance the same (20 units). As the evolutionary fitness evaluation, 4 trials were conducted, and the normalized neural entropy of the active agent was measured based on its behavior in the presence of a “ghost” partner.

G. Dynamical Systems Analysis

In order to analyze the complexity of the state-spaces for the best pair of agents in each approach, we performed the dynamical systems analysis of the autonomous continuous-time recurrent neural network (CTRNN) using *Dynamica* (version 1.0.9), a *Mathematica* package for the analysis of smooth dynamical systems, developed by Randall D. Beer.

Results

This section presents the results obtained from the best pair of agents in each of the approaches, layer-based constrained and layer-based unconstrained. These results are analyzed in detail in the Discussion and Conclusions section.

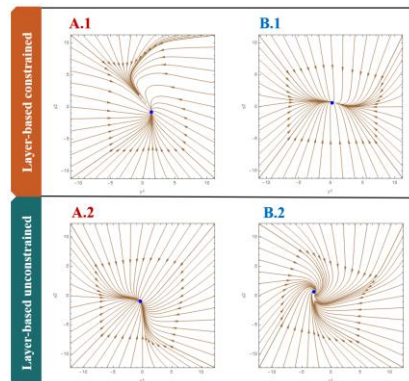


Figure 3: Dynamical systems analysis of the autonomous continuous-time recurrent neural network for the best pair of agents in the layer-based constrained (**A.1** and **B.1**) and layer-based unconstrained (**A.2** and **B.2**) approaches. The state-spaces of the dynamical systems are showing representative flow structure of a region of the activation space of the CTRNN. It can be seen that for red (**A.1** and **A.2**) and blue (**B.1** and **B.2**) agents there is a single stable fixed-point attractor (blue dot), the position of this point and its structure of attraction depend on the input values. In this case, the input values were those obtained at the end of the corresponding simulation (Generation 2000, Trial 1, best run in each case). The coordinates of the attractors are: (1.30753, -0.768629) in **A.1**, (0.2131, 0.6247) in **B.1**, (-0.42956, -0.96966) in **A.2** and (-2.9214, 0.6546) in **B.2**, where y_1 -axis represents the states of neuron 1 and y_2 -axis represents the states of neuron 2.

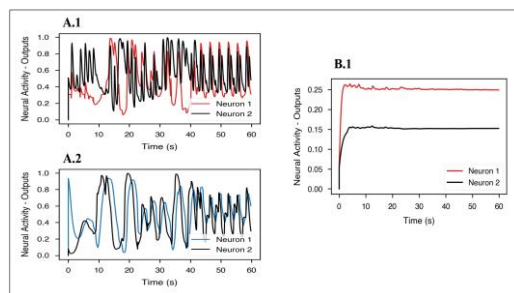


Figure 4: (**A.1** and **A.2**) Neural activity of the two neurons of red agent (**A.1**) and blue agent (**A.2**) in interactive scenario (representative trial from the best pair of agents in layer-based unconstrained approach). As it can be seen, when both agents are interacting the neural activity shows chaotic aperiodic activity that cannot be produced by 2-dimensional decoupled CTRNNs, as demonstrated in Figure 4. (**B.1**) Neural activity of the two neurons of red agent under “ghost” condition. It can be observed that when red agent is in the presence of a “ghost” partner, the neural activity demonstrates remarkably lower complexity than the neural activity of the same agent in interactive scenario (**A.1**).

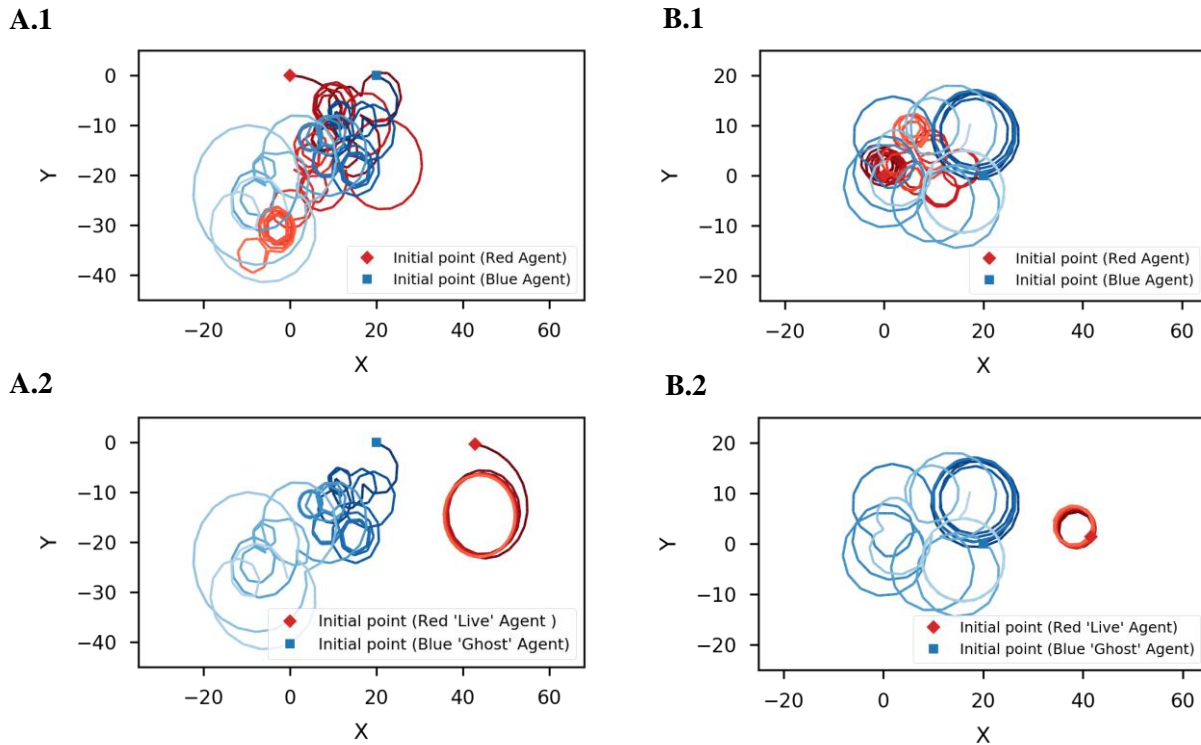


Figure 5: Results of behavioral patterns in different interaction conditions (in all images, earlier motion is darker than later motion to show directionality). **(A.1 and A.2)** Behavioral patterns for layer-based constrained approach. **(A.1)** Red and blue agents interacting in spiraling pairwise movement. **(A.2)** Red agent under “ghost” condition. In the presence of a “ghost” partner, the red agent moves in simple loops, therefore, its behavioral complexity is significantly reduced. **(B.1 and B.2)** Behavioral patterns for layer-based unconstrained approach. **(B.1)** Red and blue agents interacting in spiraling nested loops movement, where the red agent moves in smaller loops and the blue agent moves in bigger loops. **(B.2)** Red agent under “ghost” condition. In the presence of a “ghost” partner, the red agent moves in simple loops, therefore, its behavioral complexity is significantly reduced.

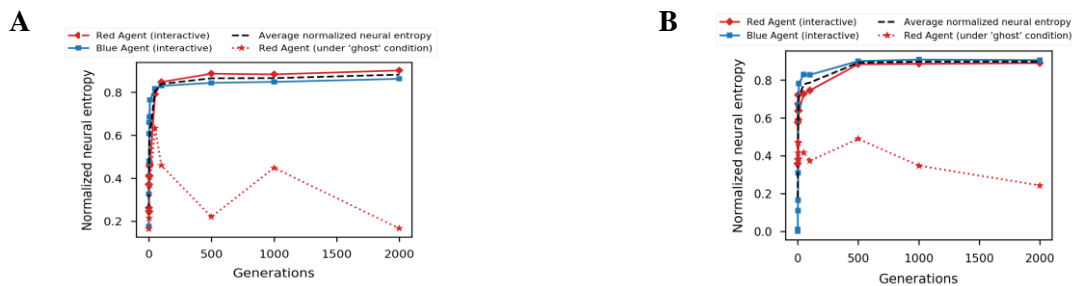


Figure 6: Normalized neural entropy of best pair of agents in each selected generation - best run (interactive vs under “ghost” condition scenarios) **(A)** Layer-based constrained approach. **(B)** Layer-based unconstrained approach. For both approaches it can be observed that when agents are able to mutually interact with each other they exhibit higher levels of normalized neural entropy, while under “ghost” condition, the “live” agent’s normalized neural entropy drops, therefore, it suffers a loss in internal complexity. The highest entropy scores of red agent under “ghost” condition are achieved during the initial generations until generation 50 (for **A**) and generation 500 (for **B**), after that they start to drop until generation 2000.

Discussion and Conclusions

Our results in Figure 3 have shown that either in the layer-based constrained or the layer-based unconstrained approaches, we got a single fixed-point attractor in the state-space analysis of the autonomous continuous-time recurrent neural networks of each agent, it follows that, what insights can we get from these findings?

Previous minimal models of adaptive behavior in different task domains that have followed evolutionary robotics methodology have gotten similar results, i.e. only single fixed-point attractors. For instance, Campos and Froese (2017) developed a minimal model on referential communication based on the waggle dance of the bee to set the task for a receiver and a sender, then, by performing dynamical system

analysis, they found only one fixed-point attractor that changed to different positions for each role, instead of having two different attractors, therefore, being an example of action switching models (Agmon and Beer, 2014). Furthermore, in social interaction minimal models, these results have been consistent as well, even when increasing the number of neurons (Froese and Fuchs, 2012) or having structurally identical pair of agents interacting, i.e. clones (Froese et al., 2013a). What is interesting to remark here is that in our findings the state-spaces also showed complementary roles of the CTRNN for red and blue agents, however, there was never specified a task to achieve, as described before for other previous models. This is one of the most insightful ideas that we got by going deeper in the Candadai et al. (2019) model: by maximizing the neural entropy in the evolutionary optimization, we were not expecting a particular behavior from the agents, however, in both approaches the strategy that the agents found was the same: by mutually interacting. At this point, it is worth recalling that elevated levels of neural entropy have been associated with improved cognitive performance. Therefore, if those elevated levels of neural entropy in the agents were achieved by mutually interacting, as shown in Figure 6, this suggests that social interaction might play a relevant role for cognition. Thus, these ideas might reject the classical view of cognitive science where cognitive complexity relies only on internal complexity.

Following the previous points, now we can understand the reason why our results showed only single fixed-point attractors. According to Zarco and Froese (2018), in “world-involving” scenarios obtained by evolutionary robotics methodology, the evolutionary search leads to a CTRNN structure that makes the agent to be interactively guided by the world. When an agent is evolved to display adaptive behavior, its CTRNN controllers usually exhibit a single attractor, however, still able of fruitful dynamics. Then, from the dynamical perspective, we can conclude that our agents in interaction became the whole brain-body-environment-body-brain system (Froese et al., 2013b) demonstrating chaotic aperiodic neural activity as shown in Figure 4 (A.1 and A.2), which in principle should require 3-dimensional decoupled CTRNNs. Thus, when an agent was in the presence of a “ghost” partner, this system was incomplete, generating consequently, a loss in neural and behavioral complexity of the “live” agent as shown in Figures 4 (B.1), 5 (A.2 and B.2) and 6. Finally, we propose thinking about how human cognition is enriched in real “world-involving” scenarios given that our daily life is full of social interactions.

In future work we will extend these results by implementing 3-neurons model and maximizing transfer entropy.

Acknowledgments

We are grateful to the Scientific Computing & Data Analysis Section at OIST for the use and support of the High-Performance Computing (HPC) resources.

Our model was implemented with the help of the Evolutionary Agents C++ software package v1.2 and analyzed using *Dynamica*, a *Mathematica* package for the analysis of smooth dynamical systems, both provided by Randall D. Beer.

We thank Maria Gohlke for helping us with Figure 1.

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